# Dynamics of electromagnetic two-stream interaction processes during longitudinal and transverse compression of an intense ion beam pulse propagating through background plasma.*

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# Motivation

- Beam pulse must be compressed transversely and longitudinally by factors $10^4$ or more to get to HIF or HEDP relevant regime.
- To avoid defocusing the space charge of the beam must be neutralized by a dense plasma.
- Impose velocity tilt to compress longitudinally.
- Use solenoidal magnet to compress transversely.
- The electrostatic two-stream instability may lead to longitudinal beam heating and could degrade the longitudinal compression of the beam pulse.
- The electromagnetic Weibel instability may cause transverse filamentation of the beam, which may degrade transverse compression.
- Fields from the focusing magnets can change the nature of collective instabilities experienced by the compressing beam.
- Therefore, it is extremely important to analyze how beam compression and external magnetic field change the instabilities.

# Outline

- Eikonal (WKB) method is discussed.

- Electrostatic two-stream instability is studied for a beam compressing

  a) transversely

  b) longitudinally

- It is shown that longitudinal compression has strong stabilizing effect on two-stream instability.

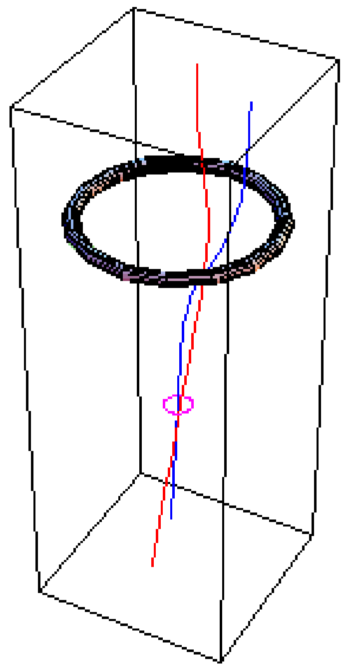
- Electromagnetic Weibel (filamentation) instability is studied for a beam compressing

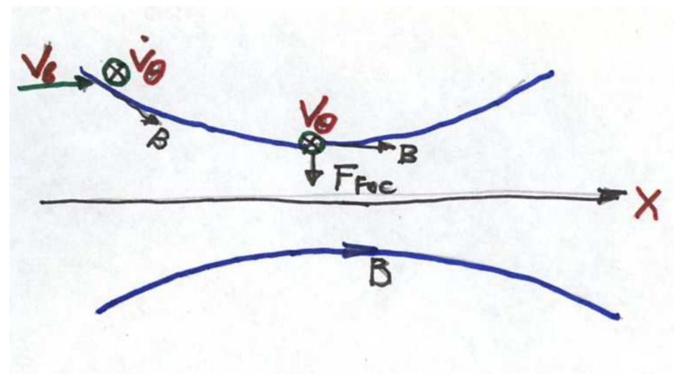
  a) transversely

  b) longitudinally

- Effects of solenoidal magnetic field on electromagnetic Weibel and electrostatic two-stream instabilities are analyzed.

# Transverse focusing of a heavy ion beam

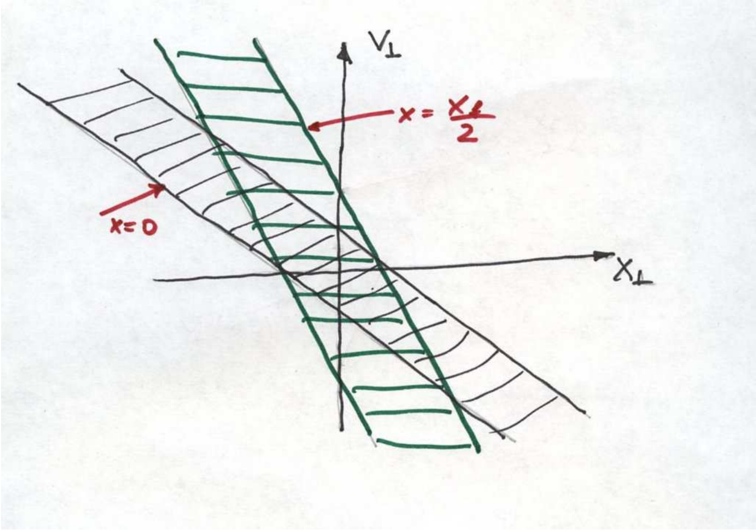





$$v_\theta = \frac{qA_\theta}{mc} = r\frac{qB_{||}}{2mc} = \frac{r\omega_{cb}}{2}.$$

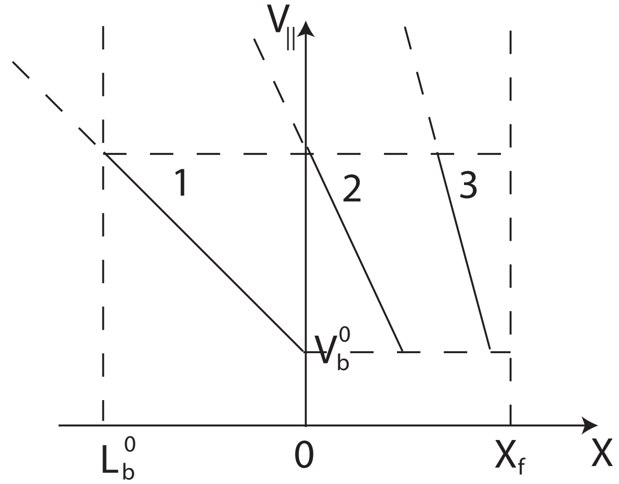
$$\frac{dv_r}{dt} = \frac{v_\theta^2}{r} - \frac{qB_{||}}{mc}v_\theta = -r\frac{\omega_{cb}^2}{4}.$$

# Transverse focusing of a heavy ion beam, cont'd




$$v_\perp = -\frac{x_\perp v_b}{X_f - x}, \quad v_{||} = v_b, \quad n_b = \frac{n_{b0}}{(1 - x/X_f)^2}, \quad v_{th\perp} \quad = \quad \frac{v_{th0}}{(1 - x/X_f)}$$

# Longitudinal focusing of a heavy ion beam



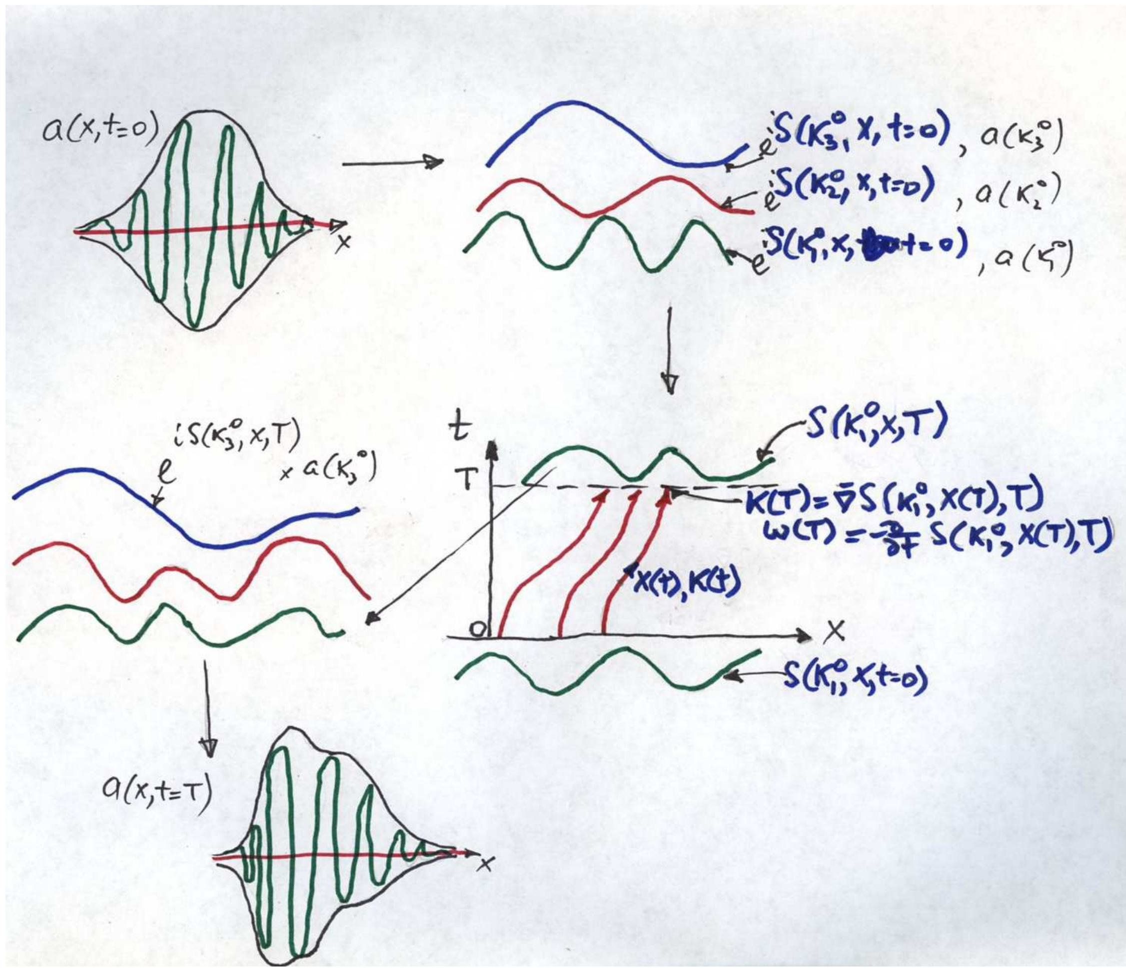

$$v_{||}(t,x) = \frac{v_b T_f - x}{T_f - t},$$

$$n_b(t) = \frac{n_{b0} T_f}{T_f - t},$$

# Eikonal (WKB) method

$$a = \int d\mathbf{k}_0 a(\mathbf{k}_0) \exp[iS(\mathbf{x}, t, \mathbf{k}_0)],$$

$$\mathbf{k} = \partial S/\partial \mathbf{x} \quad and \quad \omega = -\partial S/\partial t, \quad D(\mathbf{k}, \omega) = 0, \quad \frac{1}{k}\frac{dk}{dx} \ll k$$



# Example: no space-time dependence in any plasma parameter.

$$S = -\omega_0 t + \mathbf{k}_0 \cdot \mathbf{x}.$$

- Here $\omega_0$ and $\mathbf{k}_0$ are constants related by dispersion relation

$$D(\omega_0, \mathbf{k}_0) = 0.$$

- The amplitude is the linear combination

$$a = \int d\mathbf{k}_0 a(\mathbf{k}_0) \exp\{i[-\omega(\mathbf{k}_0)t + \mathbf{k}_0 \cdot \mathbf{x}]\}.$$

# Ray equations for the waves

- $D(\omega, \mathbf{k}) = 0$

$$\frac{\partial \mathbf{k}}{\partial t} + \left(\mathbf{v}_g \cdot \frac{\partial}{\partial \mathbf{x}}\right)\mathbf{k} \equiv \frac{d\mathbf{k}}{dt} = \frac{\partial D/\partial \mathbf{x}}{\partial D/\partial \omega}$$

$$\frac{\partial \omega}{\partial t} + \left(\mathbf{v}_g \cdot \frac{\partial \omega}{\partial \mathbf{x}}\right) \equiv \frac{d\omega}{dt} = -\frac{\partial D/\partial t}{\partial D/\partial \omega}$$

$$\mathbf{v}_g \equiv -\frac{\partial D/\partial \mathbf{k}}{\partial D/\partial \omega}.$$

$$\mathbf{k} = \partial S/\partial \mathbf{x} \quad and \quad \omega = -\partial S/\partial t.$$

# Electrostatic Two-stream Instability between the beam ions and plasma electrons

- In this case, the dispersion function $D$ is defined by

$$D = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pb}^2(t, \mathbf{x})}{[\omega - \mathbf{k} \cdot \mathbf{v}_b(t, \mathbf{x})]^2}.$$

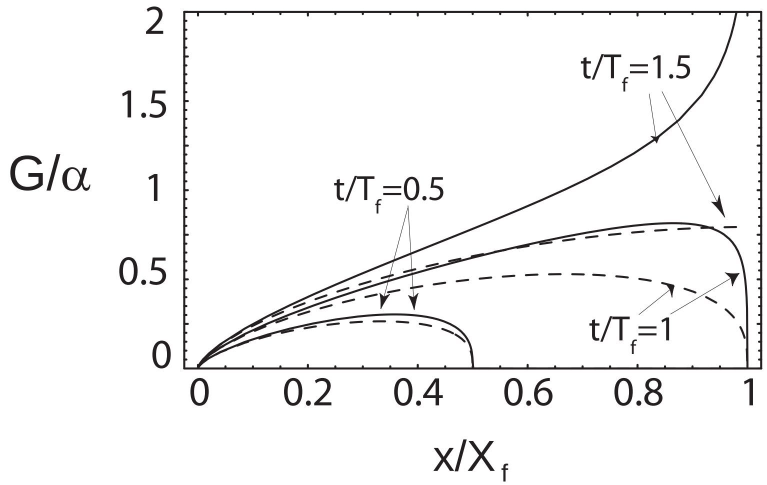
- During transverse compression

$$v_\perp = - \frac{x_\perp v_b}{X_f - x}, \quad v_{||} = v_b, \quad \omega_{pb}(x) = \frac{\omega_{pb0}}{(1 - x/X_f)^2}.$$

- During longitudinal compression

$$v_{||}(t, x) = \frac{v_b T_f - x}{T_f - t}, \quad v_\perp = 0, \quad \omega_{pb}(t) = \frac{\omega_{pb0} T_f}{T_f - t}.$$

# Two-stream instability during transverse compression

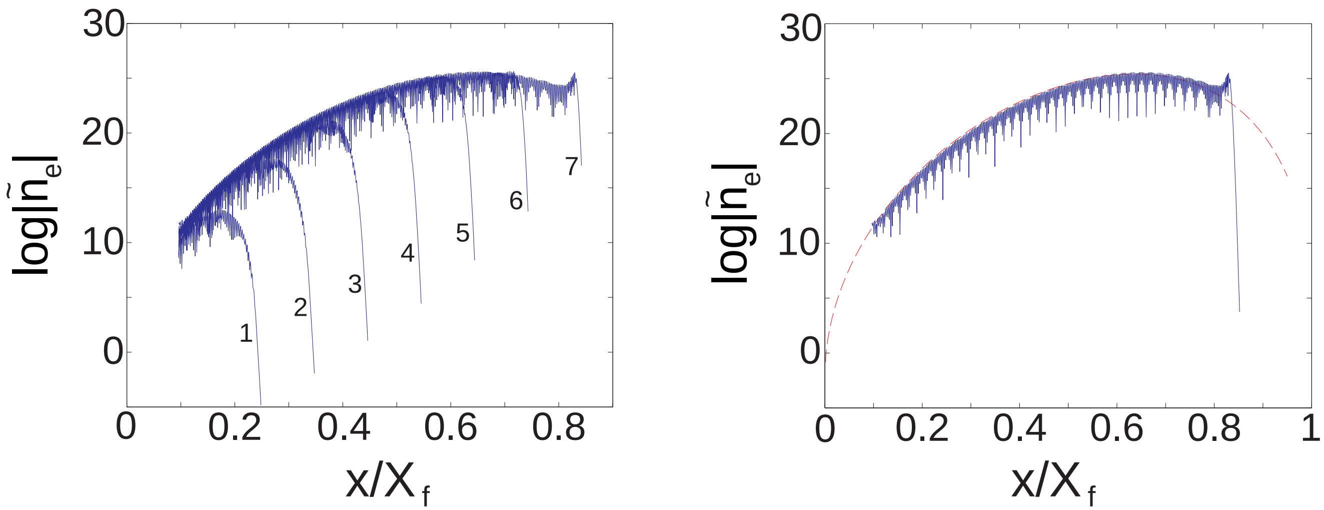



$$a \sim exp(G)$$

$$G = \alpha \left[ \ln \frac{1}{(1 - x/X_f)} \right]^{2/3} \left[ \frac{t}{T_f} - \frac{x}{X_f} \right]^{1/3}$$

$$\alpha = \frac{3\sqrt{3}}{4} T_f \omega_{pe}^{1/3} \omega_{pb0}^{2/3}$$

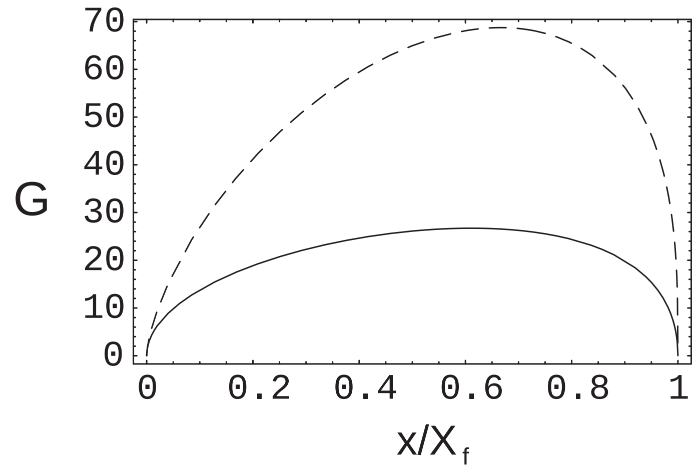
# Two-stream instability during longitudinal compression




$$|a| \sim \exp[G(x)]$$

$$G(x) = -ImS = (\omega_{pb0}T_f)\sqrt{2\left(1-\frac{x}{X_f}\right)}F\left[ArcCos\left(\sqrt{1-\frac{x}{X_f}}\right)\Big|\frac{1}{2}\right]$$

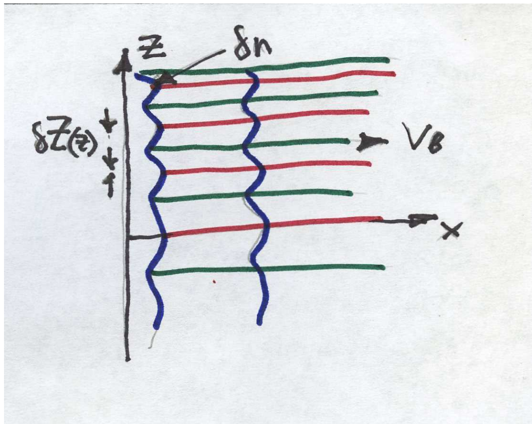
# Velocity tilt significantly reduces the growth rate




$$(\omega_{pb}^{0}/\omega_{pe})^2 = 10^{-3} \quad and \quad (\omega_{pb}^{0} T_f)^2 = 1000$$

$$G_{tilt}/G_{notilt} \sim (\omega_{pb}^{0}/\omega_{pe})^{1/3} \ll 1$$

# Weibel filamentation instability




$$\delta n = -n_0 \partial_z (\delta Z).$$

$$-\partial_z B = \frac{4\pi}{c} q v_b \delta n = -\frac{4\pi}{c} q v_b n_0 \partial_z (\delta Z).$$

$$\frac{d^2(\delta Z)}{dt^2} = \frac{q}{mc} v_b B = \left(\frac{v_b}{c}\right)^2 \left(\frac{4\pi q^2 n_0}{mc}\right) (\delta Z) = \gamma_0^2 (\delta Z)$$

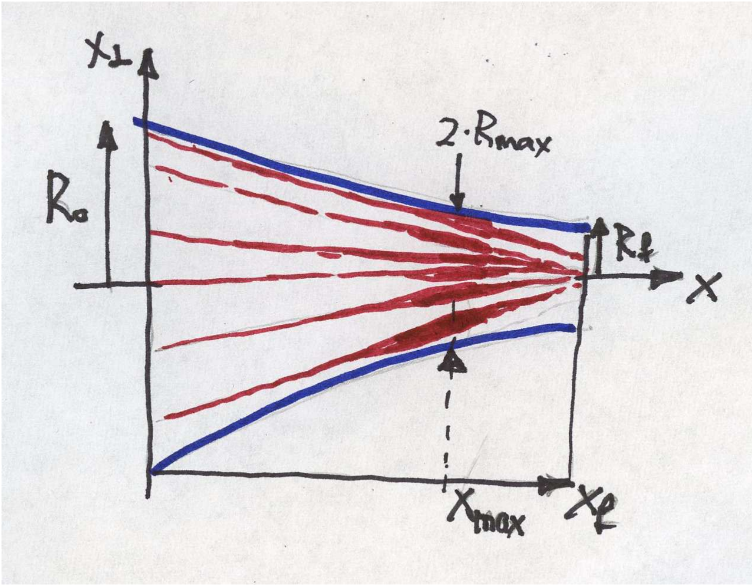
$$\omega = \mathbf{k} \cdot \mathbf{v} + i\omega_{pb} \frac{v_b}{c} - i v_{thb} k_\perp,$$

# Ion beam filamentation during transverse compression

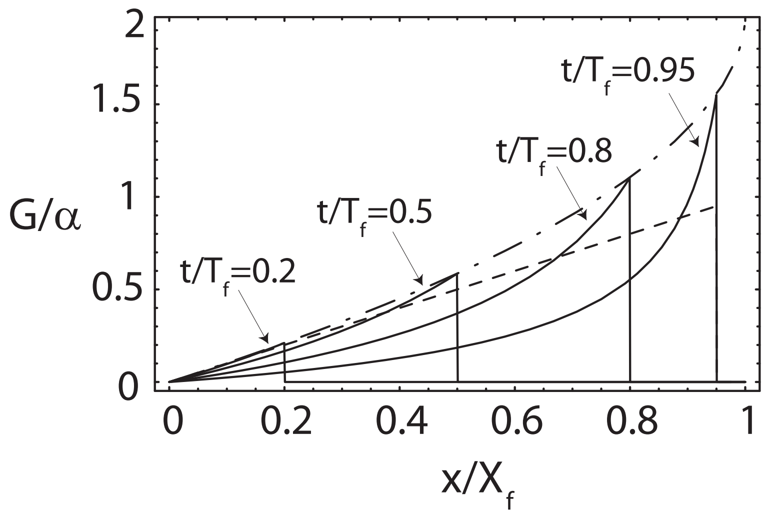
$$G_{max} \approx \alpha \ln \left[ \frac{R_0}{R_{max}} \right], \qquad where \qquad \alpha = \omega_{pb}^0 X_f / c$$

$$1 - x_{max}/X_f \sim \frac{R_{max}}{R_0}$$

$$R_{max} = R_f (R_0 / \alpha \delta_{pe})$$



# Weibel instability during longitudinal compression

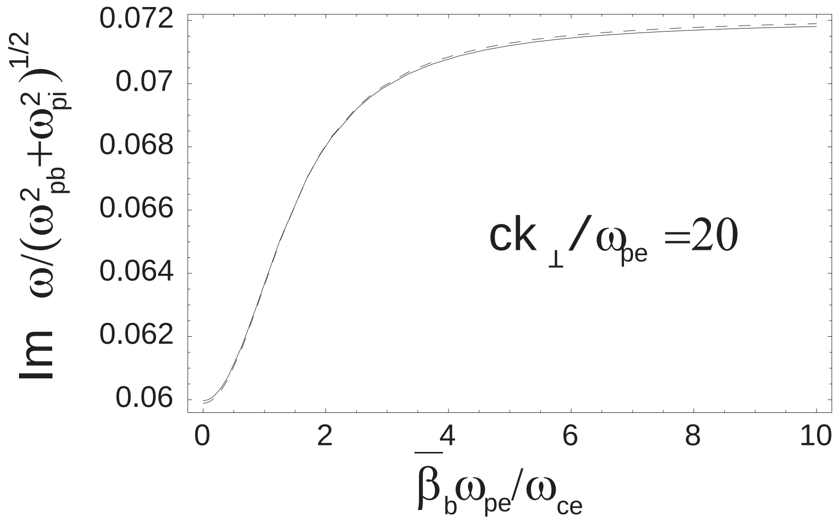



$$a \sim \exp(G)$$

$$G = \alpha \left( \frac{x}{X_f} \right) \left\{ \frac{2(1 - t/T_f)^{1/2}}{x/X_f} \left[ \frac{1}{(1 - x/X_f)^{1/2}} - 1 \right] \right\}$$

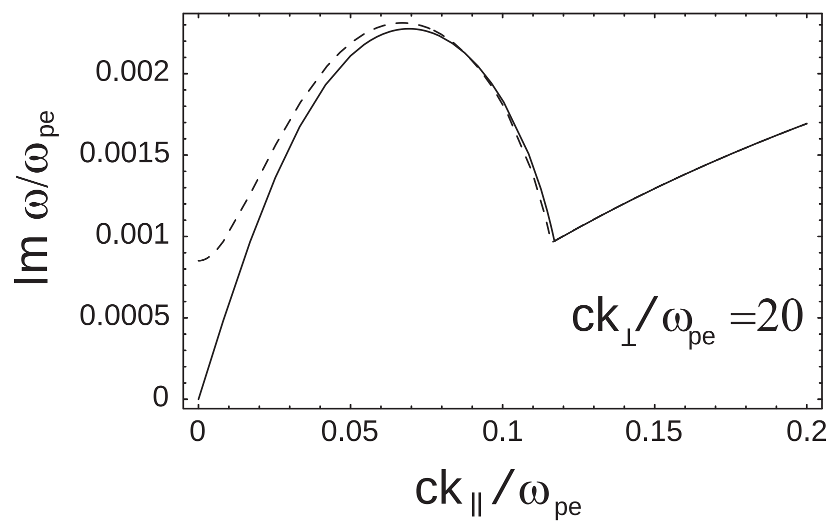
$$\alpha = \omega_{pb}^0 X_f / c$$

# Weibel instability for ion beam propagating along solenoidal magnetic field ($B_0 \neq 0$)




$$\beta_b\omega_{pe}/\omega_{ce} \ll 1$$

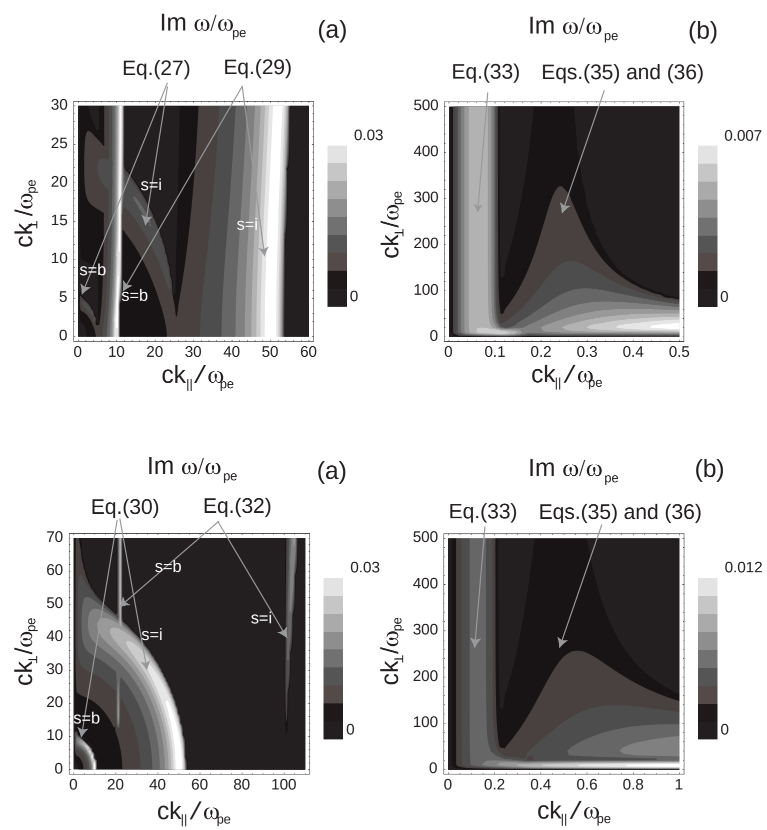
# Weibel instability is limited to long longitudinal wavelengthes




$$c^2 k_{||}^2 \ll \frac{\omega_{pb}^2 \omega_{pi}^2}{(\omega_{pb}^2 + \omega_{pi}^2)}$$

# Lower-Hybrid and Whistler waves are driven unstable by the beam $\omega \approx k_{||} v_b$

$$\omega_{LH} = \frac{\omega_{pi}}{(1 + \omega_{pe}^2/\omega_{ce}^2)^{1/2}} \qquad \omega_W = \frac{k_{||}}{k} \frac{\omega_{pe}\omega_{ce}}{(\omega_{pe}^2 + \omega_{ce}^2)^{1/2}}$$



# Conclusions

- The geometrical optics eikonal (WKB) approach to studying the space-time development of beam-plasma instabilities has been summarized.

- Two-stream instability for a radially converging beam has a much larger growth rate compared with the case of a non-converging beam.

- The longitudinal compression leads to a significant reduction in the growth rate of the two-stream instability compared with the case without an initial velocity tilt.

- The number of e-foldings proportional to the number of beam-plasma periods $1/\omega_{pb}$ during the compression time $T_f$.

- Ion beam filamentation growth during transverse compression is faster than exponential.

- Transverse thermal velocity spread limits the number of e-foldings, with maximum growth proportional to $\alpha = \omega_{pb} X_f / c$, where $X_f$ is the compression length.

# Conclusions, cont'd

- The maximum gain for filamentation instability during longitudinal compression is reached at the head of the pulse, and decreases with time after the head has passed the observation point.

- Influence of magnetic field on instability growth rates becomes significant if $\beta_b \omega_{pe}/\omega_{ce} \ll 1$.

- Electromagnetic Weibel instability (filamentation) is limited to $c^2 k_{||}^2 \ll \omega_{pb}^2 \omega_{pi}^2 / (\omega_{pb}^2 + \omega_{pi}^2)$.

- For $c^2 k_{||}^2 \gg \omega_{pb}^2 \omega_{pi}^2 / (\omega_{pb}^2 + \omega_{pi}^2)$ the instability becomes low-frequency electrostatic lower-hybrid or modified two-stream instability.